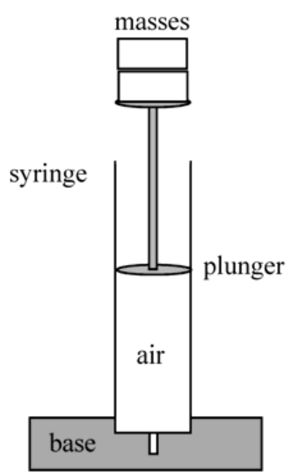
DETERMINING THE UNIVERSAL GAS CONSTANT

The general gas law relates pressure P, volume V, temperature T, the number of gram moles n, and the universal gas constant R, as $PV = nRT$. A sealed syringe with a fixed mass of gas has the volume reduced by placing masses on top of the plunger. The atmospheric pressure is P_0, the mass is m, the plunger's cross-sectional surface area is A, gravity is g and the resulting applied pressure is force over area or $\frac{mg}{A}$.

From the equation $nRT = PV = \left(P_0 + \frac{mg}{A}\right)V$ we obtain $\frac{1}{V}(RnT) = m\left(\frac{g}{A}\right) + P_0$. A graph of the reciprocal of the volume against the applied mass yields a straight line from which R is calculated. This assumes the temperature is kept constant and we determine n.

A calibrated syringe is sealed with a plunger. Masses are carefully placed on top of the plunger as shown.



DCP 1

The syringe is calibrated from zero to 35 cc in steps of 1 cc. It is estimated that the uncertainty in the measurement of volume is 0.4 cc. It is difficult to read the scale and the edge of the plunger has a noticeable width, so the uncertainty is at least ±0.4 cc even though the least count reading is 0.1 cc.

Each 500 g mass was measured on a digital balance with a precision of 0.1 g. In all cases the measured mass was less than 1 g off. A typical measure is $m_1 = 499.3\text{ g}$. The masses are thus assumed to be accurate to ±1 g or $\Delta m = \pm 0.001\text{kg}$.

Raw Data Measure	Mass m / kg $\Delta m = \pm 0.001\text{kg}$ per 500 g mass	Volume V / cm^3 $\Delta V = \pm 0.4\text{cm}^3$
1	0.000	34.6
2	0.500	33.0
3	1.000	30.0
4	1.500	26.9
5	2.000	25.1
6	2.500	23.5
7	3.000	22.0
8	3.500	20.1
9	4.000	19.0
10	4.500	17.8
11	5.000	17.0

The diameter *d* of the syringe was measured with vernier calipers and found to be $d = (2.33 \pm 0.01) \times 10^{-2}\text{m}$.

DCP 2

The area of the plunger surface is $A = \pi\left(\frac{d}{2}\right)^2 = 4.26\times10^{-4}\,\mathrm{m}^2$.

The room temperature was 16°C or 289 K. To determine n, P_0 was measured with a barometer and found to be $1.07\times10^5\,\mathrm{Pa}$.

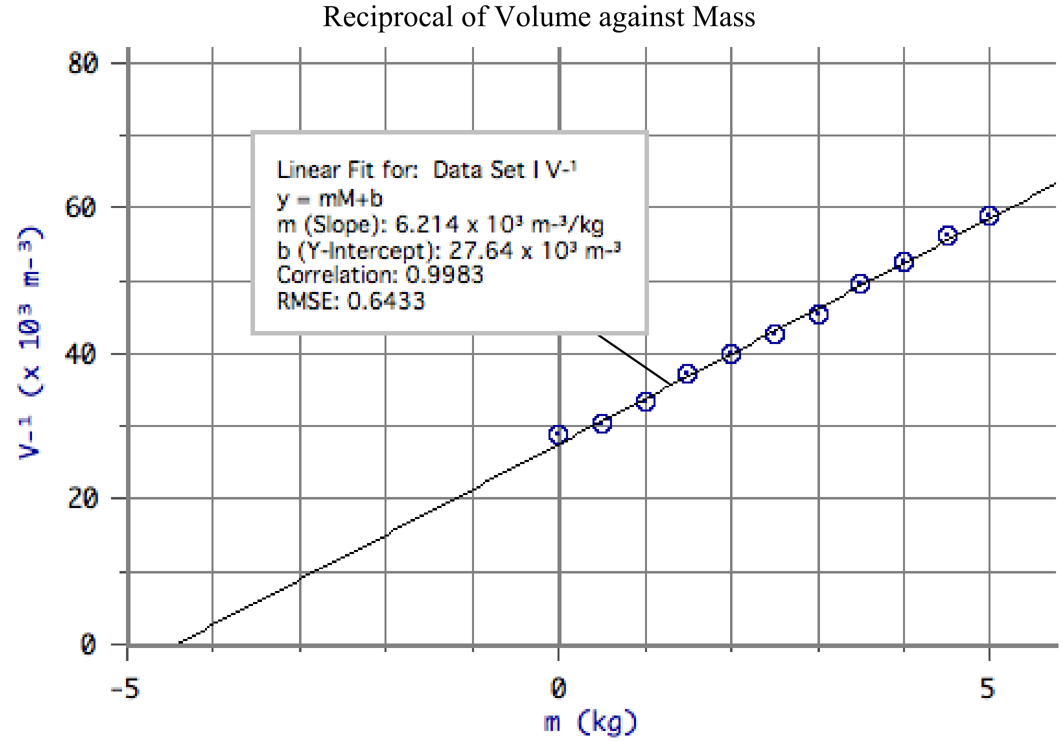
One mole of a gas at STP $\left(T = 273\mathrm{K},\ P = 1.01\times10^5\,\mathrm{Pa}\right)$ occupies $2.24\times10^{-2}\,\mathrm{m}^3$. Therefore

$$n = \frac{P_0V_0/T_0}{P_{\mathrm{STP}}V_{\mathrm{STP}}/T_{\mathrm{STP}}} = \frac{273\times1.07\times3.46\times10^{-5}}{289\times1.01\times2.24\times10^{-2}} = 1.55\times10^{-3}\,\mathrm{mol}$$

where $3.46\times10^{-5}\,\mathrm{m}^3$ is the volume of the syringe at 289K. Gravity g is assumed to be $9.81\,\mathrm{m\,s}^{-2}$.

All calculations, including the slope, were done on the spreadsheet of the graphing program *LoggerPro* 3.4.6. One example: $\frac{1}{V_1} = \frac{1}{34.6\times10^{-5}\,\mathrm{m}^3} = 2.89\times10^4\,\mathrm{m}^{-3}$.

DCP 3



Where $nRT = \left(P_0 + \frac{mg}{A}\right)V$ we find $nRT\left(\frac{1}{V}\right) = m\left(\frac{g}{A}\right) + P_0$ and then solve for R.

CE 1

The slope is $= 6.214 \times 10^3\,\mathrm{m^{-3}kg^{-1}}$ and so we find $R = \frac{g}{(\text{slope})A\,n\,T} = 8.27\,\mathrm{J\,mol^{-1}\,K^{-1}}$.

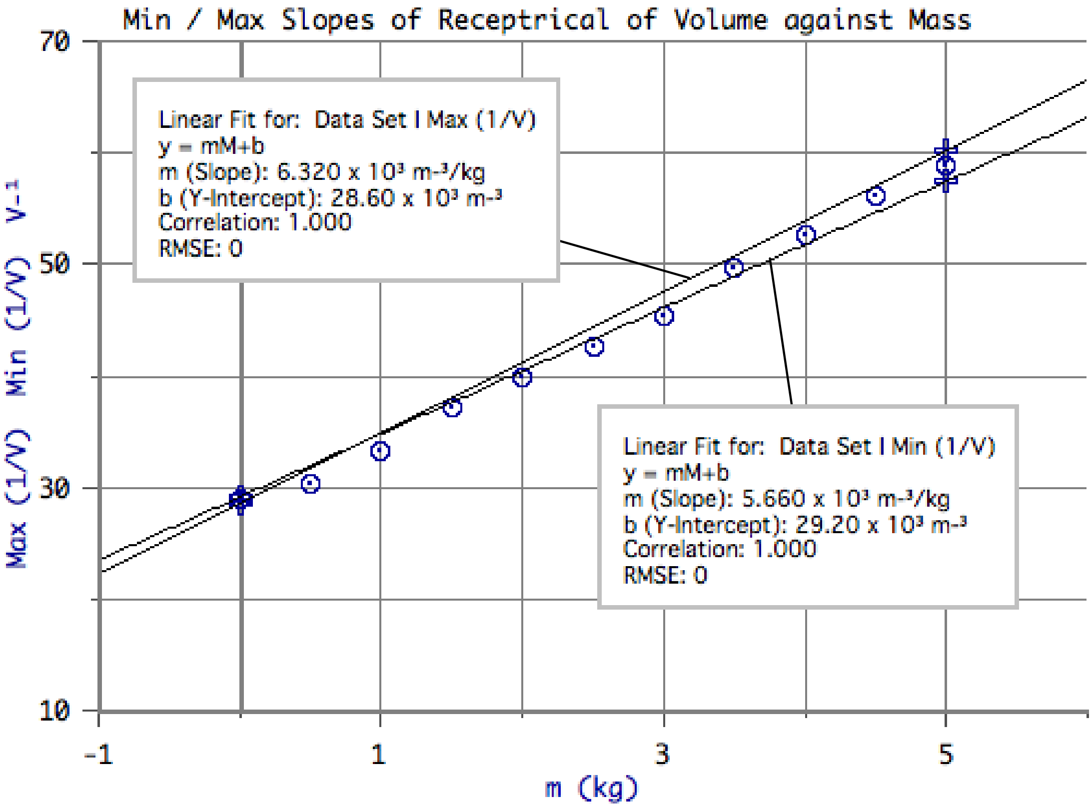
The experimental value is less than 1% off the accepted value of R, which is $8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$. However, this does not mean that the accepted value lies within the uncertainty range of the experimental value. For a correct statement of our results, in the form $R_{\mathrm{exp}} \pm \Delta R_{\mathrm{exp}}$, we need to process the uncertainties.

Here are the uncertainties in the reciprocal of the volume for the first and last data points. These are used to construct error bars on the second graph.

DCP 3

Data Number	$\frac{1}{V}(\text{max})$	$\frac{1}{V}(\text{min})$
#1 $28.9\times10^3\,\text{m}^{-3}$	$\frac{1}{V_1+\Delta V}=28.6\times10^3\,\text{m}^{-3}$	$\frac{1}{V_1-\Delta V}=29.2\times10^3\,\text{m}^{-3}$
#11 $58.8\times10^3\,\text{m}^{-3}$	$\frac{1}{V_{11}-\Delta V}=60.2\times10^3\,\text{m}^{-3}$	$\frac{1}{V_{11}+\Delta V}=57.5\times10^3\,\text{m}^{-3}$

The next graph shows the minimum and maximum slopes using the uncertainties in the volume measurements of the first and last data points.



CE 1

The maximum slope is $6.32\times10^3\,\text{m}^{-3}$ and the minimum slope is $5.66\times10^3\,\text{m}^{-3}$. The experimental range for R is thus:

$$R_{\text{Min}}=\frac{g}{(\text{slope}_{\text{Max}})A\,n\,T}=8.13\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1}$$

CE 1

$$R_{\text{Max}} = \frac{g}{\left(\text{slope}_{\text{Min}}\right) A\,n\,T} = 9.08\ \text{J mol}^{-1}\ \text{K}^{-1}$$

The uncertainty in R is $\Delta R = \pm\frac{R_{\text{Max}} - R_{\text{Min}}}{2} - \frac{9.08 - 8.13}{2}\text{mol}^{-1}\ \text{K}^{-1} = \pm 0.5\ \text{mol}^{-1}\ \text{K}^{-1}$.

Therefore the experimental value and its uncertainty is

$$R_{\text{exp}} \pm \Delta R_{\text{exp}} \approx \left(8.3 \pm 0.5\right)\text{J mol}^{-1}\text{K}^{-1}.$$

In conclusion, the measured value of R was found have an uncertainty of about $\pm 6\%$. The experimental range is from 7.8 to $8.8\ \text{J mol}^{-1}\text{K}^{-1}$ and this range includes the accepted value, where $R_{\text{accept}} = 8.3\ \text{J mol}^{-1}\text{K}^{-1}$.

CE 2

Clearly, the source of greatest error in the experiment is in the measurement of the volume. The mass uncertainty is only a fraction of one percent, the area uncertainty is about 0.4% and the temperature uncertainty is only 0.3%. None of these are significant. (One interesting note is that using the graph intercept, an experimental value for atmospheric pressure is calculated as $1.02 \times 10^{5}\ \text{Pa}$. This is about 5% off the barometer measurement.)

There is also a reliance on other data that is assumed rather than measured such as the value of g and the determination of n, and there is the problem of the plunger sticking and hence giving inaccurate readings.

CE 3

The slight but noticeable scatter of data about the best straight-line could be improved by taking more readings. Using a much larger syringe with a finer calibration scale could increase the accuracy in the volume. This would also enable more readings to be taken and this would help eliminate inaccuracies due to the plunger sticking. However, with repeated readings there is the possibility of air leaking from the syringe.

To overcome the dependency on assumed data, an alternative method is needed such that a value of R can be determined from directly measured quantities.

Finally, the first data point, where there is no applied mass, appears to be off the trend line compared to the rest of the data. Excluding this data point the graph slope gives a value of $R = 7.66\,\mathrm{J\,mol^{-1}K^{-1}}$ which is actually lower than the value of R that includes the first data point.

CE 3 Perhaps the mass of the plunger or friction between the plunger and the syringe makes a difference, but it seems insignificant here.

